| Mark Scheme 1 | Matching the syllabus written by EDEXCEL Curriculum 2004+ |
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| Calculators Allowed<br>*Where appropriate leave your answers to 3 s.f.* | |
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1. Use identity $\cos^2 A + \sin^2 A = 1 \Rightarrow \cos^2 A = 1 - \sin^2 A$ — M1
And substitute into $\cos 2x = \cos^2 x - \sin^2 x$
$\cos 2x = 1 - 2\sin^2 x$
$\sin^2 x = {}^1/_2(1 - \cos 2x)$
Separating variables: $\int 1\,dy = \int (\sin^2 x)\,dx$ — M1
Hence y = $\frac{1}{2}\int (1-\cos 2x)\,dx$
$= \frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right) + c$ — A1A1
Substitute $x = 0.1$, y = 0.2 to obtain c = 0.19966… = 0.200 (3 s.f.) — M1
i.e. y = $\frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right) + 0.200$ — A1 (6)

Note: alternative solution using integration by parts: y = $\int \sin x \sin x\,dx$

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2. a) Expand binomial $(1 + x)^n = 1 + nx + \frac{n(n-1)x^2}{1\times 2} + \frac{n(n-1)(n-2)x^3}{1\times 2\times 3} + \ldots$ — M1

$$(1-x)^{\frac{1}{2}} = 1 + \frac{1}{2}(-x) + \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)(-x)^2 + \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(\frac{1}{6}\right)(-x)^3$$ — M1

$$= 1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3$$ — A1A1(4)

b) Substitute $x = 10^{-2}$ into expansion from a) to obtain:
$\left(1 - \frac{1}{100}\right)^{\frac{1}{2}} = 0.994987\ldots$ — M1 ft
$\frac{1}{10}\sqrt{99} = 0.994987\ldots$
$\frac{3}{10}\sqrt{11} = 0.994987\ldots$ — M1
Hence $\sqrt{11} = 3.3166$ (5 s.f.) — A1 (3)

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3. a) Attempt integration by parts — M1
with $u = x$ $v' = e^x$ — M1
$\int (xe^x)\,dx = xe^x - \int (1.e^x)\,dx$ — A1A1
$= xe^x - e^x + c$ — A1 (5)

b) Attempt integration by parts twice — M1
$\int (x^2e^x)\,dx = x^2e^x - \int (2xe^x)\,dx$ — A1A1
$= x^2e^x - 2(xe^x - e^x)$ substitute answer from a) — M1
$= x^2e^x - 2xe^x + 2e^x + c$ — A1 (5)

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4. a) Use scalar product formula $\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_2$ — M1
Vectors are perpendicular $\therefore$ $\mathbf{a}.\mathbf{b} = 0$ — M1
Hence $-1 \times 1 + 1 \times c + 1 \times 1 = 0$ — M1

Answer c = 0 A1 (4)

b) Form 2 simultaneous equations:
$1 + 3\mu = 1 + \lambda$ M1
$2 + 2\mu = \lambda$ M1
Solve simultaneous equations to find $\mu = 2$ or $\lambda = 6$ M1
Substitute one of these back into the appropriate line equation to find position vector of intersection point: $7\mathbf{i} + 6\mathbf{j} + 7\mathbf{k}$ A1 (4)

c) Use formula $\mathbf{a.b} = a_1b_1 + a_2b_2 + a_3b_3 = |a||b|\cos\theta$ with $3\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{i} + \mathbf{j} + \mathbf{k}$ M1
i.e. $\cos\theta = \dfrac{(3\times1)+(2\times1)+(3\times1)}{\sqrt{3^2+2^2+3^2}\sqrt{1^2+1^2+1^2}}$ M1
$\therefore \theta = \cos^{-1}(^8/_{\sqrt{66}}) = 10.024\ldots = 10.0°$ (3 s.f.) A1 (3)

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5. a) Write as $f(x) = \dfrac{A}{x-1} + \dfrac{B}{x+2}$ M1
Therefore $x + 1 = (x + 2)A + (x - 1)B$ M1
Substitute $x = 1$, or similar method, (to find $A = \frac{2}{3}$) M1
Substitute $x = -2$, or similar method, (to find $B = \frac{1}{3}$) M1
Hence $f(x) = \dfrac{2}{3(x-1)} + \dfrac{1}{3(x+2)}$ A1A1(6)

b) $\dfrac{df}{dx} = -\dfrac{2}{3(x-1)^2} - \dfrac{1}{3(x+2)^2}$ A1A1
Substituting $x = 2$, $\dfrac{df(2)}{dx} = -\dfrac{2}{3} - \dfrac{1}{48} = -\dfrac{11}{16}$ M1A1(4)

c) $\displaystyle\int_4^5 \frac{2}{3(x-1)} + \frac{1}{3(x+2)}\,dx$
$= \left[\dfrac{2}{3}\ln(x-1) + \dfrac{1}{3}\ln(x+2)\right]_4^5$ M1M1
$= \dfrac{2}{3}\ln 4 + \dfrac{1}{3}\ln 7 - \dfrac{2}{3}\ln 3 - \dfrac{1}{3}\ln 6 = \dfrac{2}{3}\ln\left(\dfrac{4}{3}\right) + \dfrac{1}{3}\ln\left(\dfrac{7}{6}\right)$ or $= \dfrac{1}{3}\ln\left(\dfrac{56}{27}\right)$ M1A1(4)

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6. a) Substitute $x = \frac{1}{2}$ into $x^2 + y^2 + 2x + 4y + 1 = 0$ M1
Simplify to quadratic $y^2 + 4y + \frac{9}{4} = 0$ M1
Use quadratic formula to solve for y
Hence coordinates (0.5, –0.67712…) and (0.5, –3.3228…)
= (0.5, –0.68) and (0.5, –3.32) (3 s.f.) A1 (3)

b) Differentiating implicitly,
$2x + 2yy' + 2 + 4y' = 0$ A1A1A1A1
$y'(2y + 4) = -2x - 2$ M1
$\therefore y' = \dfrac{-2x-2}{2y+4} = -\dfrac{(x+1)}{y+2}$ A1 (6)

c) Substituting coordinates (0.5, –0.67…),

$y' = \dfrac{-2x-2}{2y+4} = -\dfrac{(x+1)}{y+2} = -\dfrac{3\sqrt{7}}{7} = -1.1338\ldots = -1.13$ (3 s.f.) M1A1

Substituting coordinates (0.5, –3.3…),

$y' = \dfrac{-2x-2}{2y+4} = -\dfrac{(x+1)}{y+2} = \dfrac{3\sqrt{7}}{7} = 1.1338\ldots = 1.13$ (3 s.f.) M1A1(4)

d) y = 1.1…$x$ + c

Substitute (0.5, –3.3…) to find c M1

c = – 3.8898… = 3.89 (3 s.f.)

and hence y = 1.13$x$ – 3.89 (3s.f.) A1 (2)

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7. V $= \pi\int y^2\, dx$ M1

$= \pi\int_0^{\pi} x^2 \sin x\, dx$ limits M1A1

$\int v\dfrac{du}{dx}\, dx = uv - \int u\dfrac{dv}{dx}\, dx$

$\dfrac{du}{dx} = \sin x \quad v = x^2$ M1

$u = -\cos x \quad \dfrac{dv}{dx} = 2x$ M1

V = $-\cos x \times x^2 - \int -\cos x(2x)\, dx$ A1

I $= \int 2x\cos x\, dx$ $\quad \dfrac{du}{dx} = \cos x \quad v = 2x$

$u = \sin x \quad \dfrac{dv}{dx} = 2$ M1

I $= 2x\sin x - 2\int \sin x\, dx$ M1

$= 2x\sin x + 2\cos x$ A1

Therefore V = $\pi\left[-x^2\cos x + 2x\sin x + 2\cos x\right]_0^{\pi}$ M1

$= \pi\left[-\pi^2\cos\pi + 2\pi\sin\pi + 2\cos\pi - (2\cos 0)\right]$ M1

$= \pi\left[\pi^2 - 2 - 2\right]$

$= \pi\left[\pi^2 - 4\right]$ cm$^3$ or equivalent A1 (12)

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| Calculators Allowed<br>***Where appropriate leave your answers to 3 s.f.*** | |
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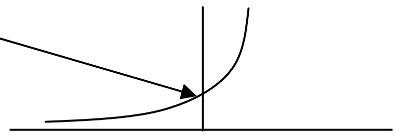
1. a) Curve sketch which cuts the y-axis at **y = 3** **A1**A1(2)



b) The area is $\int_0^{\ln 10} 3e^x \, dx = \left[3e^x\right]_0^{\ln 10} = 27$ square units M1A1M1A1

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2. a) $\int_1^4 \frac{1}{1+2x} \, dx = \int_1^4 \frac{1}{y}\frac{dx}{dy} \, dy$ M1

y = 1 + 2x, $\frac{dy}{dx} = 2, \frac{dx}{dy} = \frac{1}{2}$ M1

$\int_1^4 \frac{1}{y} \frac{dx}{dy} \, dy = \left[\frac{1}{2}\ln|1+2x|\right]_1^4 = \frac{\ln 9}{2} - \frac{\ln 3}{2} = \frac{\ln 3}{2}$ A1 (3)

b) Using the trapezium rule correctly, with n = 3. i.e. $\frac{h}{2}\left[y_0 + 2y_1 + 2y_2 + y_3\right]$ M1

Using f(1), f(2), f(3) or f(4) M1

h = 1; [$x_0 = \frac{1}{3}$; $x_1 = \frac{2}{5}$; $x_2 = \frac{3}{7}$; $x_3 = \frac{4}{9}$] A1[A1]

Estimated area = $\frac{1}{2}\left[\frac{1}{3} + \frac{4}{9} + 2\left(\frac{2}{5} + \frac{3}{7}\right)\right] = 1\frac{137}{630}$ (= 1.2174… = 1.22 square units to 3 s.f.) A1 (5)

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3. a) Using $(1 + x)_n = 1 + nx + n(n-1)x^2 + \ldots$ $|x| < 1$ M1

Substitute in for n = –3

$1 - 3x + \frac{(-3)(-4)x^2}{2} + \frac{(-3)(-4)(-5)x^5}{6}$ M1

$= 1 - 3x + \mathbf{6x^2} - 10x^3$ A1**A1**A1

(5)

b) $(4 + 4x)^{-3} = 4^{-3}(1 + x)^{-3}$ M1

$\therefore (4 + 4x)^{-3} = \frac{1}{64}[1 - 3x + 6x^2 - 10x^3]$ A1ft (2)

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4. a) t = $x$ –1 M1

Substitute in

$\therefore$ y = $(x - 1)^2 + 2$ [$= x^2 - 2x + 3$] M1A1(3)

b) Using chain rule $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$, M1

$\frac{dx}{dt} = 3\sin^2 t\cos t$ A1

Using $\frac{dt}{dx} = \frac{1}{\frac{dx}{dt}}$ M1

$\therefore \frac{dy}{dx} = \frac{1 - \sin t}{3\sin^2 t \cos t}$ A1 (4)

c) i) Gradient = $\frac{dy}{dx}$, substitute in $t = \pi/4$

$= (1 - \sin \pi/4)/(3\sin^2(\pi/4)\cos \pi/4)$ M1

$= \frac{2\sqrt{2}-2}{3} = 0.27614\ldots = 0.276$ (3 s.f.) A1 (2)

ii) gradient of normal at $t = \pi/4$ is $\frac{3}{2-2\sqrt{2}} = -3.6213\ldots = -3.62$ (3 s.f.) M1 ft

$y = \frac{3}{2-2\sqrt{2}}x + c$ ($y = -3.62x + c$) M1

When $t = \pi/4$ (=0.785…), $x = 1 + \pi/4$ (=1.785…), $y = \pi/4 + \cos \pi/4$ (= 1.492…) A1

Substitute in

$\pi/4 + \cos \pi/4 = \frac{3}{2-2\sqrt{2}}(1 + \pi/4) + c$

$c = \pi/4 + \cos \pi/4 - \frac{3}{2-2\sqrt{2}}(1 + \pi/4)$ ( = 7.9580… = 7.96 (3 s.f.)) M1 ft

Therefore equation of normal is $y = \frac{3}{2-2\sqrt{2}}(x - 1 - \pi/4) + \pi/4 + \cos \pi/4$ A1 (5)

or $y = -3.62x + 7.96$ (3 s.f.)

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5. a) $\frac{7}{(x+6)(2x+1)} = \frac{A}{(x+6)} + \frac{B}{(2x+1)}$ M1

$\therefore 7 = A(2x + 1) + B(x + 6)$ A1

Use $x = -6$

$\therefore 7 = -11A \;\; \therefore A = -\frac{7}{11}$ M1A1

Use $x = -\frac{1}{2}$

$\therefore 7 = B \times \frac{11}{2} \;\; \therefore B = \frac{14}{11}$ M1A1

$\therefore \frac{7}{(x+6)(2x+1)} = \frac{14}{11(2x+1)} - \frac{7}{11(x+6)}$ A1 (7)

b) $f'(x) = \frac{7}{11(x+6)^2} - \frac{28}{11(2x+1)^2}$ A1A1

$f'(1) = \frac{7}{539} - \frac{28}{99} = -\frac{17}{63}$ M1A1(4)

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6. a) $r_1 - r_2$ M1

$\overrightarrow{AB} = 2\mathbf{i} + 0\mathbf{j} - 6\mathbf{k}$ A1A1(3)

b) Calculate Modulus = $\sqrt{2^2 + 6^2} = \sqrt{40}$ M1

$\frac{1}{\sqrt{40}}[2\mathbf{i} - 6\mathbf{k}]$ or $\frac{1}{\sqrt{10}}[\mathbf{i} - 3\mathbf{k}]$ A1 (2)

c) $-\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda[2\mathbf{i} - 6\mathbf{k}]$ or $\mathbf{i} + 2\mathbf{j} - 5\mathbf{k} + \lambda[2\mathbf{i} - 6\mathbf{k}]$ A1A1(2)

d) Using $\mathbf{a}.\mathbf{b} = ab\cos\theta = a_1b_1 + a_2b_2 + a_3b_3$ M1

Substitute in $\sqrt{40}\sqrt{30}\cos\theta = 2 + 30 = \mathbf{32}$ *A1***A1**

$\therefore \cos\theta = \dfrac{32}{20\sqrt{3}}$ M1

$\therefore \theta = 22.517\ldots = 22.5°$ (3 s.f.) A1 (5)

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7. a) $\int \sin x \cos x \, dx = \frac{1}{2}\int \sin 2x \, dx$ M1

$= \frac{1}{2} \times \frac{-\cos 2x}{2} = -\frac{\cos 2x}{4}$ A1

Apply limits $= [0] - \left[-\frac{1}{4}\right] = \frac{1}{4}$ M1A1(4)

b) $I = \int e^x \sin x \, dx$, where $u = \sin x$ $v' = e^x$

$u' = \cos x$ $\mathbf{v = e^x}$ M1**M1**

$\therefore I = e^x \sin x - \int e^x \cos x \, dx$ A1

Let $J = \int e^x \cos x \, dx$ and repeat integration, where $u = \cos x$ $v' = e^x$

$u' = -\sin x$ $v = e^x$ M1

$J = e^x \cos x - \int -e^x \sin x \, dx$

$= e^x \cos x + I$

$\therefore I = e^x \sin x - [e^x \cos x + I]$ M1

$\therefore I = \frac{1}{2} e^x(\sin x - \cos x) + \mathbf{k}$ A2**A1**(8)

c) $x\sin x = y\cos y$

$\frac{d}{dx}(x \sin x) = \frac{dy}{dx} \times \frac{d}{dy}(y \cos y)$ M1

attempt to use product rule M1

$\sin x + x \cos x = \frac{dy}{dx}[\cos y - y \sin y]$ A1A1

$\therefore \frac{dy}{dx} = \frac{\sin x + x \cos x}{\cos y - y \sin y}$ A1 (5)

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| Calculators Allowed<br>***Where appropriate leave your answers to 3 s.f.*** | |
|  | **Core Mathematics – C4** |

1\. a) The curve is an *inverted exponential* which crosses the y-axis at **y = 1** and the $x$-axis at $x = \ln(3/2) \approx 0.405\ldots$ — *M1***A1** A1 (3)

b) $\int_2^3 (3 - 2e^x)\,dx$ — M1

$= [3x - 2e^x]_2^3$ — A1A1

Substituting in limits; — M1

$= (9 - 2e^3) - (6 - 2e^2) = 3 - 2e^3 + 2e^2 = -22.392\ldots$

$\therefore$ Area = –22.392… = 22.4 (3 s.f.) below $x$-axis — A1 (5)

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2\. a) Use binomial theorem with suitable substitution — M1

$$(1-4x)^{1/2} = 1 + \left(\frac{1}{2}\right)(-4x) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(-4x)^2 + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}(-4x)^3$$ — A1

$= 1 - 2x - \mathbf{2x^2} - \mathbf{4x^3}$ — A1*A1***A1**(5)

b) $$\left(1 - 4\times\frac{1}{100}\right)^{1/2} = \left(100 - \frac{4}{100}\right)^{1/2} = \frac{\sqrt{96}}{10}$$ — M1

$$= \frac{4\sqrt{6}}{10}$$ — A1

$$\frac{4\sqrt{6}}{10} = 1 - 2\times\frac{1}{100} - 2\times\frac{1}{10000} - 4\times\frac{1}{1000000} = 0.979796$$ — M1

$$\therefore \sqrt{6} = \frac{10}{4}\times 0.979796$$

$$\therefore \sqrt{6} = 2.44949 = 2.4495 \text{ (5 s.f.)}$$ — A1 (4)

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3\. a) Using chain rule $\frac{dy}{dx} = \frac{dy}{dt}\times\frac{dt}{dx}$, where $\frac{dx}{dt} = 2\sin t\cos t$ and $\frac{dy}{dt} = -\sin t$ — M1*A1*A1

$$\therefore \frac{dy}{dx} = -\frac{\sin t}{2\sin t\cos t} = -\frac{1}{2\cos t}$$ — A1 (4)

b) $$-\frac{1}{2\cos \pi/4} = A\sqrt{2}$$ — M1

$$-\frac{1}{2\sqrt{2}} = A\sqrt{2}$$

$$\therefore A = -\frac{1}{2}$$ — A1 (2)

c) $x = \sin^2 t$ $\quad y = 1 + \cos t$

$x = 1 - \cos^2 t$ $\quad \therefore \cos t = y - 1$ — M1

$\cos^2 t = (y - 1)^2$ — A1

Using $\sin^2 t + \cos^2 t = 1$

$x = 1 - (y - 1)^2$ or further simplified i.e. $x = 2y - y^2$ — A1 (3)

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4\. a) $u = \cos x$

$\frac{du}{dx} = -\sin x$ A1

$\therefore dx = -\frac{1}{\sin x}\ du$

$\sin x\cos^3 x\ dx \Rightarrow \int u^3 \sin x \times \left(-\frac{1}{\sin x}\right) du$ M1

$= \int -u^3\ du$ A1

$= -\frac{1}{4}u^4 + c$ A1

$= -\frac{1}{4}\cos^4 x + c$ A1 (5)

b) $\sin A + \sin B \equiv 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ M1

$\frac{A+B}{2} = 6x \Rightarrow A + B = 12x$

$\frac{A-B}{2} = 5x \Rightarrow A - B = 10x$

$\therefore 2A = 22x$ M1

$A = 11x$

$B = x$

Thus $2\sin x\cos 5x = \sin 11x + \sin x$ A1 (3)

c) $\int \sin 6x \cos 5x\ dx$

$= \int \frac{1}{2}\left(\sin 11x + \sin x\right) dx$ M1

$= -\frac{1}{22}\cos 11x - \frac{1}{2}\cos x + c$ A1A1(3)

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5. a) $\frac{x+1}{(x-1)^2(x+2)} \equiv \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+2)}$ M1

$x + 1 = A(x-1)(x+2) + B(x+2) + C(x-1)^2$

When $x = 1$, $2 = B \times 3$ M1

$\therefore B = \frac{2}{3}$ A1

When $x = -2$, $-1 = C \times (-3) \times 2$

$\therefore C = -\frac{1}{9}$ A1

Since $0 = A + C$

$A = \frac{1}{9}$ A1

$\therefore \frac{x+1}{(x-1)^2(x+2)} = \frac{1}{9(x-1)} + \frac{2}{3(x-1)^2} - \frac{1}{9(x+2)}$ A1 (6)

b) $\frac{d}{dx}f(x) = -\frac{1}{9(x-1)^2} - \frac{4}{3(x-1)^3} + \frac{1}{9(x+2)^2}$ A1

$\frac{d}{dx}f(2) = -\frac{1}{9\times 1} - \frac{4}{3\times 1} + \frac{1}{9\times 16}$

$= -1\frac{7}{16}$ A1 (2)

c) Area = $\int_4^5 f(x)\,dx$ M1

$$\int_4^5\left[\frac{1}{9(x-1)}+\frac{2}{3(x-1)^2}-\frac{1}{9(x+2)}\right]dx$$

$$=\frac{1}{9}\ln|x-1|-\frac{2}{3(x-1)}-\frac{1}{9}\ln|x+2|\Big|_4^5$$ A1A1A1

$$=\left(\frac{1}{9}\ln 4-\frac{1}{6}-\frac{1}{9}\ln 7\right)-\left(\frac{1}{9}\ln 3-\frac{2}{9}-\frac{1}{9}\ln 6\right)$$

$$=\frac{1}{9}\ln\left(\frac{4}{7}\right)-\frac{1}{6}-\frac{1}{9}\ln\left(\frac{1}{2}\right)+\frac{2}{9}$$

$$=\frac{1}{9}\ln\left(\frac{8}{7}\right)+\frac{1}{18}$$ A1 (5)

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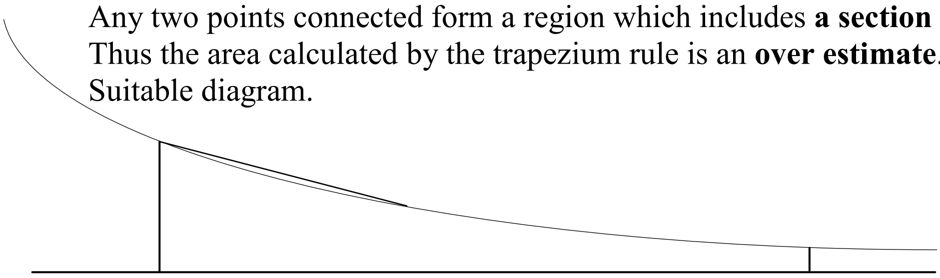
6. a) $\int_1^4 e^x\,dx=\left[e^x\right]_1^4=e^4-e^1=51.8799..$ = 51.9 or better **correctly substituting limits** M1**A1***A1*(3)

b) i) **h = 1**; $y_0 = e^{-1}$; $y_1 = e^{-2}$; $y_2 = e^{-3}$; $y_3 = e^{-4}$; **M1**M1

Area = $\frac{1}{2}[e^{-1}+e^{-4}+2(e^{-2}+e^{-3})]$ = 0.37821… = 0.378 (3 s.f.) M1A1(4)

ii) In each interval an estimate of the area of the region is obtained by joining successive points on the curve with a straight line.
Any two points connected form a region which includes **a section <u>above</u> the line**.
Thus the area calculated by the trapezium rule is an **over estimate**.
Suitable diagram.



M1

To improve accuracy, need to take more intervals. A1 (2)

c) These two graphs are reflections of each other in the y-axis A1 (1)

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7. a) $\overrightarrow{AB}$ = (i + 2j + k) – (–i – 2j + k) M1
$\overrightarrow{AB}$ = 2i + 4j A1 (2)

b) Magnitude $=\sqrt{(-1)^2+(-2)^2+1^2}$ M1A1
$=\sqrt{1+4+1}$
$=\sqrt{6}$ A1 (3)

$\overrightarrow{OA}$ = $^1/_{\sqrt{6}}$ (–i –2j + k)

c) Distance $=\sqrt{2^2+4^2}$ M1
$=\sqrt{20}$ units
$=2\sqrt{5}$ units A1 (2)

d) **i** + μ3**i** = 3**i** + λ**i**

$\mathbf{i}(1 + 3\mu) = \mathbf{i}(3 + \lambda)$

$\therefore 1 + 3\mu = 3 + \lambda \quad \leftarrow$ M1

$2j + \mu 2j = 2j + \lambda 2j$

$j(2 + 2\mu) = j(2 + 2\lambda)$ M1

$\therefore \mu = \lambda$

Substitute into $\leftarrow$

$1 + 3\lambda = 3 + \lambda$

$\therefore 2\lambda = 2$

$\lambda = 1 \Rightarrow \mu = 1$ A1

$\therefore (3 + \mu)k = (1 + A\lambda)k$

$\therefore 3 + 1 = 1 + A$

$\therefore A = 3$ A1 (4)

e) $a_1b_1 + a_2b_2 + a_3b_3 = (1)(3) + (2)(2) + (1)(3)$ M1

$= 10$ A1

$|a| = \sqrt{9+4+1} = \sqrt{14}$

$|b| = \sqrt{1+4+9} = \sqrt{14}$

**14**$\cos\theta = 10$ **B1**

$\cos\theta = \frac{5}{7} = 44.415\ldots = 44.4°$ (3 s.f.) A1 (4)

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1. $x^2 = 9\cos^2 2t \qquad y^2 = 9\sin^2 2t$
squaring and adding — M1
$\therefore x^2 + y^2 = 9\cos^2 2t + 9\sin^2 2t = 9(\underline{\cos^2 2t + \sin^2 2t}) = 9$ — <u>M1</u>A1
$\therefore$ **Circle**, with <u>centre (0, 0)</u>, <u>*radius 3*</u> — **A1**<u>A1*A1*</u>(6)

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2. $\int dy = \int (x + \sin x)\, dx$ — M1

y = $\int (x + \sin x)\, dx$ — A1

$= \dfrac{x^2}{2} - \cos x + c$ — A1

Sub in 0.1 and 0.2 to work out c — M1

$\therefore y = \dfrac{x^2}{2} - \cos x + 1.1900...$ = 1.19 (3 s.f.) — A1 (5)

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3. a) Using binomial expansion — M1

$$(1-2x)^{½} = 1 + \left(\frac{1}{2}\right)(-2x) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(-2x)^2 + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}(-2x)^3$$ M1

$$= 1 - x - \frac{1}{2}x^2 - \frac{1}{2}x^3$$ A1A1(4)

b) Sub in: $1 - 0.01 - \dfrac{1}{2}(0.01)^2 - \dfrac{1}{2}(0.01)^3 = 0.9899495$ — M1

equating $\therefore \left(1 - \dfrac{2}{100}\right)^{\frac{1}{2}} = 0.9899495$ — M1

$\therefore \dfrac{1}{10}(98)^{\frac{1}{2}} = 0.9899495$

$\therefore \dfrac{7}{10}\sqrt{2} = 0.9899495$ — M1

$\therefore \sqrt{2} = \left(\dfrac{10}{7}\right) \times 0.9899495 = 1.41421...$ = 1.4142 (5 s.f.) — A1 (4)

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4. a) Integration by parts — M1

I = $\int x \sin x\, dx \qquad u = x \quad v' = \sin x \qquad u' = 1 \quad v = -\cos x$ — M1A1

$\therefore$ I = $-x\cos x - \int (-\cos x) \times 1\, dx$
$= -\boldsymbol{x}\ \mathbf{cos}\ \boldsymbol{x} + \sin x + c$ — **A1**A1(5)

b) $I_2 = \int x^2 \cos x\, dx \qquad u = x^2 \quad v' = \cos x \qquad u' = 2x \quad v = \sin x$ — M1A1

$I_2 = x^2 \sin x - \int 2x \sin x\, dx = x^2 \sin x - \mathbf{2I}$ — **M1**

$= \boldsymbol{x^2} \textbf{ sin } \boldsymbol{x} + 2x \cos x - 2\sin x + k$ **A1**A1(5)

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5. a) $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ M1

$\frac{dy}{dt} = -2\cos t \sin t, \quad \frac{dx}{dt} = -6\sin 6t$ A1A1

Using $\frac{dt}{dx} = \frac{1}{\frac{dx}{dt}}$ M1

$\frac{dy}{dx} = \frac{-2\cos t \sin t}{-6\sin 6t} = \frac{\cos t \sin t}{3\sin 6t}$ A1 (5)

b) Substitute into equation above to find the gradient M1

i.e. Gradient = $\frac{\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}}{3 \times -1} = -\frac{1}{6}$ A1 (2)

c) $m_1 m_2 = -1 \qquad \therefore m_2 = 6$ A1 ft

$y = 6x + c$

Sub in t to find that $x = 0$ and $y = \frac{1}{2}$ and sub into $y = 6x + c$ M1

$\therefore c = \frac{1}{2} \qquad \therefore y = 6x + \frac{1}{2}$ A1 (3)

---

6. a) $\int \frac{x^3}{x^4+1} dx = \ln\left|x^4 + 1\right| + k$ OR $\ln\left|k(x^4+1)\right|$ (A1 for missing k) A2 (2)

b) $\int_1^5 x^{\frac{1}{2}} + e^x \text{dx} = \left[\frac{2x^{\frac{3}{2}}}{3} + e^x\right]_1^5$ A1A1

Substitute in limits correctly; $= \left[\frac{2 \times 5^{\frac{3}{2}}}{3} + e^5\right] - \left[\frac{2}{3} + e\right]$ M1

$= 155.86\ldots - 3.3849\ldots$

$= 152.48\ldots = 152.5$ (1 d.p.) A1 (4)

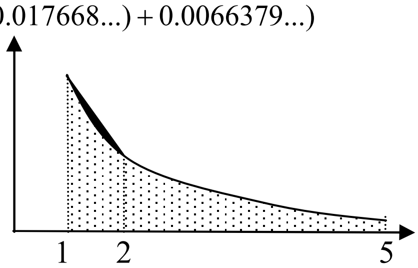
c) $h = 1, n = 4 \Rightarrow A = \frac{1}{2}(y_0 + 2(y_1 + y_2 + y_3) + y_4)$ M1

$\int_1^5 \frac{1}{x^{\frac{1}{2}} + e^x} dx \approx \frac{1}{2}(f(1) + 2(f(2) + f(3) + f(4)) + f(5))$ M1

$= \frac{1}{2}(0.26894\ldots + 2(0.11359\ldots + 0.045834\ldots + 0.017668\ldots) + 0.0066379\ldots)$ A1

$= 0.31488\ldots = 0.315$ (3 s.f.) A1

The trapezium rule gives an **over**-estimate of the integral. **A1**




Shading between curve and line segment. M1 ft(6)

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7. a) $\mathbf{a} \cdot \mathbf{b} = |a||b|\cos\theta = a_1b_1 + a_2b_2 + a_3b_3$ M1

$|a| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3} \quad |b| = \sqrt{3^2 + 2^2 + 2^2} = \sqrt{17}$ A1

$\therefore\ \cos\theta = \dfrac{-3+2+2}{\sqrt{3}\sqrt{17}} = 0.14002...$ A1

$\therefore\ \theta = 81.9505\ldots° = 81.951$ (3 d.p.) A1 (4)

b) $|AB| = \sqrt{4^2+1^2+1^2} = \sqrt{18}$ M1A1(2)

c) $\overrightarrow{AC} = (c+1)i + 4k \qquad \overrightarrow{AB} = 4i + j + k$ A1A1

$\left(\overrightarrow{AC}\right) . \left(\overrightarrow{AB}\right) = \underline{4(c+1) + 0 + 4} = \mathbf{0}$ <u>M1</u>**A1 ft**

$4c + 8 = 0 \quad \therefore\ c = -2$ A1 (5)

d) $-i + j + k$ **(or other valid position)** + $\lambda(4i + j + k)$ **<u>(or multiple, including –1)</u>** **A1**<u>A1</u>(2)

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8. a) $f(x) = \dfrac{12x}{(x+1)(2x+1)} = \dfrac{A}{x+1} + \dfrac{B}{2x+1}$ M1

$12x = A(2x+1) + B(x+1)$

$x = -1 \Rightarrow -12 = -A \Rightarrow A = 12 \qquad x = -0.5 \Rightarrow -6 = 0.5B \Rightarrow B = -12$

$\therefore\ f(x) = \dfrac{12}{(x+1)} - \dfrac{12}{(2x+1)}$ A1A1(3)

b) $f'(x) = -\dfrac{12}{(x+1)^2} + \dfrac{24}{(2x+1)^2}$ A1ftA1ft

(2)

c) $f(x) = 12\left[(x+1)^{-1} - (2x+1)^{-1}\right]$ M1

$(x+1)^{-1} = 1 - x + x^2 + \ldots \qquad -1 < x < 1$ A1

$(2x+1)^{-1} = 1 + (-1)(2x) + \dfrac{(-1)(-2)}{2}(2x)^2 + \ldots$ M1

$= 1 - 2x + 4x^2 + \ldots \qquad -0.5 < x < 0.5$ A1

$\therefore\ f(x) = 12[x - 3x^2] \qquad -0.5 < x < 0.5$ A1A1(6)

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1. Using $V = \pi\int y^2\ dx$ M1

$y^2 = \left(e^{\frac{x}{2}}\right)^2 = e^x$ A1

$= \pi\int_2^3 e^x\ dx$

$= \pi\left[e^x\right]_2^3$ A1

Substitute in limits M1

$= \pi\ (e^3 - e^2)$ A1

(5)

---

2. Separate variables $\Rightarrow \int dy = \int (x + e^x)dx$ M1

$\therefore y = \int\ (x+e^x)\ dx \qquad \Rightarrow \qquad y = \frac{x^2}{2} + e^x + c$ A1A1

Sub in $x = 1$, $y = 2$ M1

$\therefore\ c = \frac{3}{2} - e = -1.2182... = -1.22$ (3 s.f.) A1 (5)

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3. a) $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ M1

$\frac{dy}{dt} = 3\cos 3t$, $\frac{dx}{dt} = \cos t$ A1A1

$\therefore \frac{dy}{dx} = \frac{3\cos 3t}{\cos t}$ A1 (4)

b) Sub in $t = \frac{\pi}{4}$ M1

$\therefore\ \frac{dy}{dx} = -3$ A1 (2)

c) $y = -3x + c$ M1

When $t = \frac{\pi}{4}$, $x = \frac{1}{\sqrt{2}}$, $y = \frac{1}{\sqrt{2}}$ M1

$\therefore \frac{1}{\sqrt{2}} = \frac{-3}{\sqrt{2}} + c \quad \therefore c = \frac{4}{\sqrt{2}}$ or $2\sqrt{2}$ (= 2.8284… = 2.83 (3 s.f.)) A1 (3)

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4. a) $I = \int\ (3\sin 6x - \sin 2x)\ dx\ = -\frac{1}{2}\cos 6x + \frac{1}{2}\cos 2x + \mathbf{C}$ M1A1A1**A1**

(4)

b) $u = \sin x \qquad \frac{du}{\cos x} = dx$ sub this in M1 ft

$\therefore$ I = 16 ∫ $\mathbf{u^4}$ **du** $= \frac{16u^5}{5} = \frac{16\sin^5 x}{5}$ **A1**A1

Sub in limits $\therefore$ $I = \frac{1}{10}$ A1 (4)

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5. a) $D^2 = (b-1)^2 + (b+1)^2 + 12^2$ A1

Equate to $14^2$ M1

$2b^2 + 146 = 196 \quad \therefore b = +5$ or $\underline{-5}$ **A1**<u>A1</u>(4)

b) $a \cdot c = 2 - 1 - 2 = -1$ A1

$|a| = \sqrt{3}$, $|c| = \sqrt{9} = 3$ A1A1

a . c = **(a)(c)cos θ** = $\underline{a_1c_1 + a_2c_2 + a_3c_3}$ **M1**<u>M1</u>

$\cos\theta = \dfrac{-1}{3\sqrt{3}}$ A1

$\therefore$ Acute angle = 180° – θ = 78.904… = 78.9° (3 s.f.) A1 (7)

c) <u>**i + j + k**</u> **(or other correct position)** + $\lambda(\overrightarrow{AC})$ (or multiple including –1) <u>**A1**</u>A1

= **i** + **j** + **k** + λ(**i** – 2**j** – 3**k**)

i = 17 $\therefore \lambda = 16$ M1

for the **j** we have: 1+(–2×16)=–31 so consistent and so can pass through that point B1

$\therefore$ position is 17**i** – 31**j** – 47**k** i.e. D = 47 A1 (5)

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6. a) i) Multiply equations for same coefficients:

$3x = 6\cos t$

$2y = 6\sin t$ M1

Square both sides to give

$9x^2 = 36\cos^2 t$

$4y^2 = 36\sin^2 t$ M1

$9x^2 + 4y^2 = 36\cos^2 t + 36\sin^2 t$ M1

Using $\sin^2 t + \cos^2 t = 1$, Cartesian equation is $9x^2 + 4y^2 = 36$

M1A1(5)

ii) Differentiate implicitly M1

$18x + 8y\dfrac{dy}{dx} = 0$ A1A1A1

$\dfrac{dy}{dx} = -\dfrac{9x}{4y}$ A1 (5)

b) i) $\dfrac{dy}{dx} = \dfrac{dy}{dt} \times \dfrac{dt}{dx}$ $\left[\dfrac{dt}{dx} = \dfrac{1}{\left(\dfrac{dx}{dt}\right)}\right]$ M1M1

$\dfrac{dy}{dt} = 3\cos t$, $\dfrac{dx}{dt} = -2\sin t$

$\dfrac{dy}{dx} = -\dfrac{3\cos t}{2\sin t}$ or $-\dfrac{3}{2}\cot t$ A1 (3)

ii) $\dfrac{dy}{dx} = -\dfrac{3 \times \frac{x}{2}}{2 \times \frac{y}{3}} = -\dfrac{9x}{4y}$, same as aii). M1A1(2)

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7. a) $\dfrac{36x}{(2x+1)(x+2)} \equiv \dfrac{A}{2x+1} + \dfrac{B}{x+2}$ M1

$36x \equiv A(x+2) + B(2x+1)$ M1

Let $x = -2 \quad \therefore -72 = -3B \qquad B = 24$ A1

Let $x = 0 \quad \therefore 0 = 2A + B \qquad A = -12$ M1A1

$\therefore \dfrac{36x}{(2x+1)(x+2)} = -\dfrac{12}{2x+1} + \dfrac{24}{x+2}$ A1 (6)

b) $f(x) = 12[-(2x + 1)^{-1} + 2(x + 2)^{-1}]$ M1

Using binomial expansion, M1

$(2x + 1)^{-1} = 1 + (-1)(2x) + \dfrac{(-1)(-2)(2x)^2}{2} + \ldots$ M1

$= 1 - 2x + 4x^2 + \ldots \quad -1 < 2x < 1$ A1

$(x + 2)^{-1} \equiv (2^{-1})\left(\dfrac{x}{2}+1\right)^{-1}$ M1

$\left(\dfrac{x}{2}+1\right)^{-1} = 1 + (-1)\left(\dfrac{x}{2}\right) + \dfrac{(-1)(-2)\left(\dfrac{x}{2}\right)^2}{2} + \ldots = 1 - \dfrac{x}{2} + \dfrac{x^2}{4} \qquad -1 < \dfrac{x}{2} < 1$ M1A1B1

$\therefore f(x) = 12[-(1 - 2x + 4x^2) + \dfrac{1}{2} \times 2\left(1-\dfrac{x}{2}+\dfrac{x^2}{4}\right)] = 12(\dfrac{3}{2}x - \dfrac{15}{4}x^2) = \mathbf{18x} \underline{- 45x^2}$ **A1**A1

for $-\dfrac{1}{2} < x < \dfrac{1}{2}$ A1 (11)

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(75)